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Physics 34100 Applied Differential Equations
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Problem Set 7

Question 1. *Space-time Pictures of Solutions:*

(1) Isoclines:
  (a) What is the direction of the slope along the isocline $f(t, y) = s$?
  (b) Find the equation $f(t, y) = s$ for the isoclines of the following equations:
    (i) $5y' + 3y + 2 = 0$
    (ii) $y' + ty = 0$
    (iii) $y' = y - y^3$
    (iv) $y' = -cos^2(t)y$
(2) Sketch a few isoclines for each of the previous equations. Include slope zero isoclines.
(3) Add arrows to each isocline in the direction of the slope. Include extra slope arrows to fill out the diagram.
(4) Add a few solution lines by "connecting" arrows.
(5) Use the space-time flow field and the solution to describe the conditions for solutions to exist in the following equations:
  (a) $y' = y/t$.
  (b) $y' = t/y$.

Question 2. *Phase-space Pictures:*

(1) Consider the following linear odes.
  (a) $y'' - 2y' - 3y = 0$
  (b) $y'' + 3y' + 2y = 0$
  (c) $y'' + y = 0$
  (d) $y'' + 2y' + 3y = 0$
(2) Convert each to a system of first-order equations for y and y'.
(3) Convert each to a matrix equation for $\vec{y} = \begin{bmatrix} y & y' \end{bmatrix}^T = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$ and find the companion matrix A.
(4) Assume $\vec{y}(t) = e^{\lambda t}\vec{x}$ where $\vec{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and find all λ's.
(5) For each λ find the special direction $\vec{x}$, such that $A\vec{x} = \lambda\vec{x}$. Note: $c\vec{x}$ also in the special direction, so only one component of $\vec{x}$ is independent. Often a simple way to find $\vec{x}$ is to set either x_1 of x_2 to 1.
(6) Use these results to define $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ and $S = \begin{bmatrix} \vec{x}_1 & \vec{x}_2 \end{bmatrix}$ and show that $AS = S\Lambda$.
(7) Sketch the phase diagram for each and identify the type of stability: source, sink, saddle, center, spiral source, or spiral sink.

Question 3. *Stability:*

(1) Consider the following systems:
  (a) $y' = 2y + 3z + 4y^2 + 5z^2$ and $z' = 6z + 7yz$.
  (b) $y' = 1 - yz$ and $z' = y - z^3$.
(2) Find the fixed points Y.
(3) Find the Jacobian $J(Y)$ at each fixed point.
(4) Determine if the fixed points are stable or unstable.
(5) Sketch the phase-space flow for each equation.