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Physics 34100 Applied Differential Equations
April 24, 2026

Problem Set 6

Question 1. *Laplace Transform:*

$f(t)$	$y'(t)$	$y''(t)$	t^n	e^{at}	$\delta(t-T)$	$cos(wt)$	$sin(\omega t)$	$t^n e^{\omega t}$
$F(s)$	$sY(s) - y(0)$	$s^2Y(s) - sy(0) - y'(0)$	$\frac{n!}{s}$	$\frac{1}{s-a}$	e^{sT}	$\frac{s}{s^2+\omega^2}$	$\frac{w}{s^2+\omega^2}$	$\frac{n!}{(s-a)^{n+1}}$

TABLE 1. Laplace transform pairs.

(1) Use the Laplace transform to find $Y(s)$, the poles of $Y(s)$, and the solution $y(t)$ for each of the following equations with $y(0) = 0$ and $y'(0) = 1$.
   (a) $y'' + 0y' + 4y = 0$. (undamped)
   (b) $y'' + 1y' + 4y = 0$. (under-damped)
   (c) $y'' + 2y' + 4y = 0$. (critically damped)
   (d) $y'' + 3y' + 4y = 0$. (over-damped)

(2) (a) Find the Laplace transform $Y(s)$ for $y'' = e^{at}$ with $y(0) = M$ and $y'(0) = N$.
   (b) Use partial fractions to express $Y(s)$ as

$$Y(s) = \frac{A}{s} + \frac{B}{s-a} + \frac{C}{(s-a)^2}.$$

   (c) Find $y(t)$.
   (d) Show that $y'' = e^{at}$, $y(0) = M$, and $y'(0) = N$.

(3) Find the Laplace transform of the following:
   (a) $e^{at} + e^{bt}$
   (b) $2\sinh(at) = e^{at} - e^{-at}$
   (c) $\frac{d}{dt}(2\sinh(at))$
   (d) $t\sin(t)$

(4) Find the *inverse* Laplace transform of the following:
   (a) $\frac{1}{(s-a)(s-b)}$
   (b) $\frac{s}{(s-a)(s-b)}$
   (c) $\frac{1}{s^3 - s}$

Question 2. *Numeric Solutions:*

(1) Use MATLAB to solve $y' = 1 + y^2$ with $y(0) = 0$ for $0 \geq t \geq 2$.
  (a) Plot the solution.
  (b) Does the solution make sense?
  (c) Use the analytic solution to explain any problems you see.
  (d) Re-plot the solution for any range of times that make sense.

(2) Use `ode23` and `ode45` to calculate π using:

$$\int_0^1 \frac{4}{1+t^2}\, dt = \pi.$$

  (a) What ordinary differential equation has this integral as its solution?
  (b) How does the error compare for each method, `ode23` and `ode45`?
  (c) Use `ode4` and `ode2` to find a step size h that gives similar errors to those of `ode45`.

(3) Solve:

$$y'' + \beta y' + y^3 = 0.$$

with $y(0) = 10$ and $y'(0) = 0$. Make a plot of $y(t)$ and $y'(t)$ vs t and a plot of $y'(t)$ vs $y(t)$ using a time range that includes about 10-20 oscillation for the following:
  (a) $\beta = 0$.
  (b) $\beta = -1/3$.
  (c) $\beta = 1/3$.
  (d) $\beta = 100(1 - y^2)$.
  (e) $\beta = -\sin(2t)$.

(4) Solve:

$$y'' + y^3 = e^{-t/2}.$$

with $y(0) = 1$ and $y'(0) = 0$. Make a plot of $y(t)$ and $y'(t)$ vs t and a plot of $y'(t)$ vs $y(t)$ using a time range long enough to reach steady-state.