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Physics 34100 Applied Differential Equations
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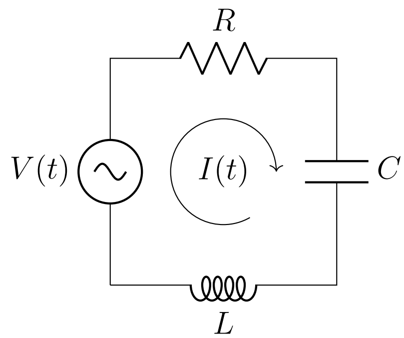
Problem Set 5



Question 1. *Driven RCL circuit:*
The general equation for an RCL circuit is

$$L\frac{dI(t)}{dt} + RI(t) + \frac{1}{C}\int_0^t I(s)\,ds = V(t)$$

(1) Rewrite this equation for $Q(t)$ where $I(t) = \frac{dQ(t)}{dt}$ to obtain a second-order differential equation in $Q(t)$. (Hint: Use the fundamental theorem of calculus.)
(2) Differentiate this equation with respect to t and obtain a second-order differential equation for $I(t)$.
(3) Find the null solution for the current $I(t)$ when $V(t) = 0$.
(4) Under what conditions on R, C, L will the null solution be a transient. How long will it take for the null solution to reach $\frac{1}{e}$ of its original value?
(5) Assuming the null solution is a transient, find the current $I(t)$ for $V(t) = V_0 e^{iwt}$.
(6) Show that if $L = t$, $R = 1$, and $C = t$ then the null solutions are $I_1(t) = sin(log(t))$ and $I_2(t) = cos(log(t))$.
(7) Find the Wronskian of the null solutions.
(8) Use these null solutions to find a solution with $L = t$, $R = 1$, and $C = t$ for a general voltage source $V(t)$. The solution can be left as an integral. When does the solution exist?

Question 2. *Method of undetermined coefficients* Find a particular solution by inspection or using the method of undetermined coefficients.

(1) $y'' + y = 3$
(2) $y'' + y' = 3$
(3) $y'' + y' + y = e^t$
(4) $y'' + y' + y = e^{st}$
(5) $y'' + y' + y = t^3$
(6) $y'' + y = sin(t)$
(7) $y'' + y = t + e^t$
(8) $y'' + 16y = e^{3t}$
(9) $y'' + 16y = te^{3t}$
(10) $y'' + y' = t^2 + 1$
(11) $y'' + 2y = tcos(t)$
(12) $y'' + y = e^{it}$
(13) $y'' - 4y' + 3y = e^t$

Question 3. *Method of variation of parameters*

(1) Find the two null solutions y_1 and y_2 for $y'' + y' - 6y = 0$.

(2) Find a particular solution using the method of variation of parameters for the following:
   (a) $y'' + y' - 6y = e^t$
   (b) $y'' + y' - 6y = e^{-t}$

(3) Using the general formula for variation of parameters:

$$y_p(t) = -y_1(t) \int_0^t \frac{y_2(T)f(T)}{W(T)}\, dT + y_2(t) \int_0^t \frac{y_1(T)f(T)}{W(T)}\, dT$$

where

$$W(t) = y_1(t)y_2'(t) - y_2(t)y_1'(t)$$

what are the initial conditions for $y_p(t)$?