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Physics 34100 Applied Differential Equations
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# Problem Set 4

**Question 1.** *Logistic Equation:*

(1) For this problem you will need the MATLAB ode solvers that we discussed in class: `ode1.m`, `ode2.m`, and `ode4.m`. Download `odex.zip` and unzip into a directory that you can find in MATLAB.

(2) Create a MATLAB function to solve the Logistic Equation:

$$y' = ay - by^2$$

(a) A function begins with the keyword `function`, then a comma separated list of outputs enclosed in brackets `[out1,out2,...,outN]`, followed by an equals sign, the name of the function and a comma separated list of inputs in parentheses `(in1,in2,...,inN)`.

```
function [out1,out2,...,outN]=function_name(in1,in2,...,inN)
```

Create a file called `logistic.m` using the command `edit logistic` in the same directory as you unziped `odex.zip`.

(b) Add the following to the file:

```
function [y,t,h]=logistic(a,b,t0,h,tf,y0,odex)
% logistic <Solves and plot the Logistic equation.>
% Usage:: [y,t,h]=logistic(a,b,t0,h,tf,y0,odex)
%
% Outputs:
% y is a column vector list containing the solution at the points t=(t0:h:tf)';
% t ??
% h is a handle to the plots h(1) numeric h(2) exact h(3) carrying capacity
%
% Inputs:
%
% a is the cooefficient of the linear growth
% b ??
% ??
% ??
% ??
% ??
% odex is the name of the ode solver e.g., ode4

% revision history:
% 03/09/2026 Mark D. Shattuck <mds> logistic.m

%% Main
```

(c) Fill in the list and descriptions of each input and output variable in place of the `??`.

(d) Next add the definition of the logistic equation `F(t,y)` using an autonomous function:

$$y' = ay - by^2 = F(t, y)$$

For example, `F=@(t,y)a*y^2` will create a function `F(t,y)` that return `a*y^2`. Test this on the command line using the following:

```
>> a=4
a =
     4
>> F=@(t,y) a*y^2
F =
    @(t,y)a*y^2
>> F(0,3)
ans =
    36
>> a*3^2
ans =
    36
>>
```

Add a line defining `F` after the comment `%% Main`.

(e) Next we need to add the solution as we discussed in class:

```
[y,t]=odex(F,t0,h,tf,y0);   % solve logistic equation using eg: odex=@ode4
```

(f) Next test the function:

```
>> [y,t]=logistic(2,3,0,1/2,6,1/300,@ode4);
>> plot(t,y)
```

These commands should create a plot showing the "S"-curve solution of the logistic equation. The final input into `logistic(...,@ode4)`, `@ode4`, is how to specify a function stored as a file like `ode4.m`. In `logistic.m` the last input `logistic(a,b,t0,h,tf,y0,odex)`, `odex`, will equal `@ode4`.

(g) Now we need to add plotting commands. Here is example code for the whole function including plotting:

```
function [y,t,h]=logistic(a,b,t0,h,tf,y0,odex)
% logistic <Solves and plot the Logistic equation.>
% Usage:: [y,t,h]=logistic(a,b,t0,h,tf,y0,odex)
%
% Outputs:
% y is a column vector list containing the solution at the points t=(t0:h:tf)';
% t ??
% h is a handle to the plots h(1) numeric h(2) exact h(3) carrying capacity
%
% Inputs:
%
% a is the cooefficient of the linear growth
% b ??
```

```
% ??
% ??
% ??
% ??
% odex is the name of the ode solver e.g., ode4

% revision history:
% 03/09/2026 Mark D. Shattuck <mds> logistic.m

%% Main

F=@(t,y) ??;          % define logistic equation

[y,t]=odex(F,t0,h,tf,y0);    % solve logistic equation
                             % using eg: odex=@ode4

%% Ploting

h=zeros(3);  % handles to the 3 plots
             % handle can be used to change plots
             % set(h(1),'color','r') would change plot 1
             % color to red.

h(1)=plot(t,y,'o-');        % plot numeric solution y vs. t
set(h(1),'linewidth',2);    % change linewidth
set(h(1),'markersize',12); % change markersize

hold all;            % add new plots to current plot

% exact solution:
tt=t0+(0:100)/100*(tf-t0);     % always use 100 points

d=??;           % constant for exact soln
ye=a./(d*exp(-a*tt)+b);
h(2)=plot(tt,ye,'k','linewidth',2);

% plot of carrying capacity
h(3)=plot(tt,0*tt+a/b,'k--');   % plot y=a/b

% labels
set(gca,'fontsize',20);
xlabel('time');
ylabel('y(t)');

hold off; % clear before adding new plot
```

This code will solve and plot the `odex` solution, the exact solution, and the carrying capacity. It contains `??` from above. So be sure to replace those.

(h) To finish the exact solution section we need the null solution constant. In class we found that

(1) $$y(t) = \frac{a}{de^{-at} + b}$$

solves the equation. Show that (1) solves the logistic equation.

(i) Find `d` for $y(0) =$`y0` and add to the code.

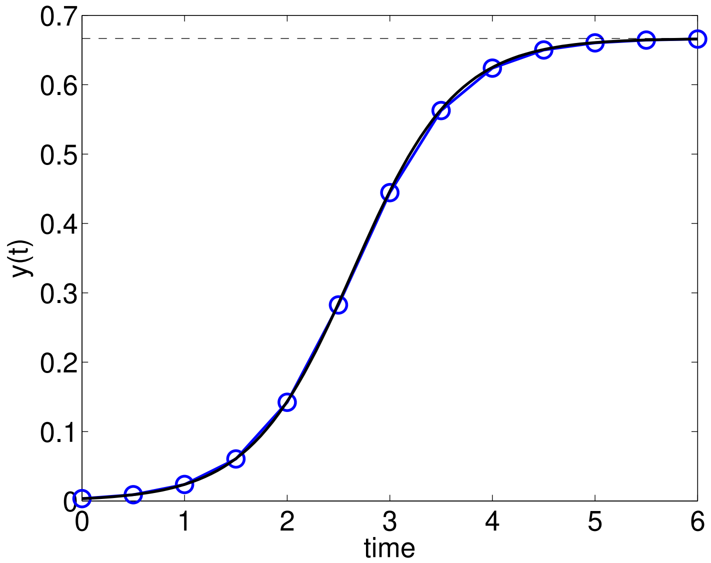


FIGURE 1. Result of `logistic(2,3,0,1/2,6,1/300,@ode4);`.

(j) When everything is working the command `logistic(2,3,0,1/2,6,1/300,@ode4);` should produce the following plot.

LaTeX note: To add a MATLAB plot to a LaTeX document use the MATLAB command

```
print -depsc2 log.eps
```

to create a color eps file. In LaTeX use:

```
\begin{figure}
    \centering
    \includegraphics[width=0.5\linewidth]{log.eps}
    \caption{Result of \mcode{logistic(2,3,0,1/2,6,1/300,@ode4);}.}
    \label{fig:logsol}
\end{figure}
```

(3) Use `logistic.m` to compare `ode4.m`, `ode2.m`, and `ode1.m`.

**Question 2.** *Variable Coefficients:*

(1) Solve the following equations for $y(t)$. You can leave your answer in terms of an integral if needed (but explain your reason).
   (a) $y' = 3t - 2ty$ given $y(0) = 1$.
   (b) $y' = 3t - 2ty$ given $y(0) = 3/2$.
   (c) $y' = e^{-t} - 2ty$ given $y(0) = 1$.
   (d) $y' = sin(t)y$ given $y(0) = 1$.
   (e) $y' = sin(t)y + t$ given $y(0) = 1$.
   (f) $y' = log(t)y$ given $y(0) = 1$.

(2) Use `ode4` to solve each equation above. Pick one and plot the exact solution and the solution from `ode4`.

**Question 3.** *General First-order Equations:*

(1) Solve the following equations for $y(t)$. You can leave your answer in terms of an integral if needed (but explain your reason).
  (a) $y' = y^2 - y^3$ given $y(0) = 1/3$.
  (b) $y' = 2y - y^3$ given $y(0) = 1/2$.
  (c) $y' = \sqrt{y}$ given $y(0) = 0$.
  (d) $y' = sin(t)y$ given $y(0) = 1$.
  (e) $y' = 3ty^2$ given $y(0) = 1$.
  (f) $y' = \dfrac{t}{log(y)}$ given $y(0) = 1$.

(2) Use `ode4` to solve each equation above. Pick one and plot the exact solution and the solution from `ode4`.

**Question 4.** *Stability:*

(1) Find the fixed points and stability of Q3(1a)-(1c).

(2) Choose one of Q3(1a)-(1c) and plot the exact solution, the `ode4` solution, and the fixed points. Add `ode4` solutions for several initial conditions in each stability region.

**Question 5.** *Second-order Equations:*

(1) Solve the following equations for $y(t)$.
  (a) $y'' + y = 0$ given $y(0) = 0$, $y'(0) = 1$.
  (b) $y'' + 4y = sin(3t)$ given $y(0) = 1$, $y'(0) = 0$.
  (c) $2y'' + y' + 3y = 0$ given $y(0) = 0$, $y'(0) = 1$.

(2) For each equation find the characteristic equation and the roots.

(3) Categorize each equation as over-damped, under-damped, or undamped.