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Physics 34100 Applied Differential Equations
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Problem Set 2

Question 1. *Introduction to Differential Equations:*

(1) Draw the graph of $y1 = e^{3t}$ and $y2 = 3e^t$ on the same axes. Which function is larger at $t = 0$? Which function is larger at $t = 1$?

(2) $\frac{d^2}{dt^2}e^t = e^t$. So $y = Ce^t$ is a solution, but a second order differential equation should have two solution. What is another solution?

(3) Find the slope of $y = e^{-t}$ at $t = 0$. Find the slope of $\frac{dy}{dt}$ at $t = 1$.

(4) Show that $\frac{C}{1-Ct}$ solves $y' = y^2$. What is the solution if $y(0) = 1$?

(5) Describe these differential equations (e.g., first-order, linear, constant coefficient).
  (a) $y' = 1$.
  (b) $y'' - y' - \sin(t) = 0$.
  (c) $(y')^2 - 3ty' - \sin(t) = 0$.
  (d) $y') - \sin(t) = 0$.
  (e) $cos(t)y' + y^2 - \sin(t) = 0$.
  (f) $y' + t^2 * y = 0$.
  (g) $y' + 3y + t = 0$.

Question 2. *Calculus Review:*

(1) What is the Taylor expansion of $y(x) = \frac{1}{1-x}$ near $x = 0$? Using the expansion show that $y'(x) = 1 + 2x + 3x^2 + \ldots = (\frac{1}{1-x})^2$.

(2) Find:

$$\frac{d}{dx}\int_0^x 4\sin(e^{3y})\tanh(y^3)\,dy.$$

Question 3. *Exponential Growth:*

(1) You start with $y(0) = \$2000$ in the bank, which grows according to the equation $y' = 0.03y$ until t=10, and then according to the equation $y' = 0.05y$ until $t = 20$. Find a piecewise function:

$$y(t) = \begin{cases} y_1(t) & 0 \le t \le 10 \\ y_2(t) & 10 \le t \le 20 \end{cases}$$

describing the growth. How much do you have when $t = 20$?

(2) Repeat the calculation for $Y_0 = \$2000$, and discrete growth where each period n 3% of Y_{n-1} is added to Y_{n-1} to get Y_n (i.e., $Y_n = Y_{n-1} + 0.03Y_{n-1} = 1.03Y_{n-1}$) or as a difference equation $Y_n - Y_{n-1} = 0.03Y_{n-1}$ for $n = 1, \ldots, 10$ (`1:10` in MATLAB) followed by $Y_n - Y_{n-1} = 0.05Y_{n-1}$ for $n = 11, 12, \ldots, 20$ (`11:20` in MATLAB). The discrete function (list or sequence) in MATLAB will look like: `[Y0 Y1 Y2]` where `Y0=2000`, `Y1` and `Y2` are `1x10` matrices representing the solutions for `n1=1:10` and `n2=11:20`. How much do you have when $n = 20$?

(3) Plot both solutions on the same graph. MATLAB code follows to get you started. You must supply the solution in place ????. If you have an equation for `y1=????` you can use the MATLAB keyword `end` e.g., `y1(end)` to get the last value of `y1`.

```
%% Continuous Growth
t1=0:.01:10;  % time for y_1(t) continous solution for period 1
t2=10:.01:20; % time for y_2(t) continous solution for period 2
t=[t1 t2];    % totaltime [concatenate]
y0=2000;      % initial condition
y1=????;    % you find solution for period 1
y2=????;    % you find solution for period 2
y=[y1 y2];    % total solution [concatenate]

%% Discrete Growth
n1=1:10;      % steps for Y1 discrete solution for period 1
n2=11:20;     % steps for Y2 discrete solution for period 2
n=[0 n1 n2];  % totaltime [concatenate]
Y0=2000;      % initial condition
Y1=????;    % you find solution for period 1
Y2=????;    % you find solution for period 2
Y=[Y0 Y1 Y2]; % total solution [concatenate]

%% Plot
h=plot(t,y/1000,n,Y/1000,'o'); % plot results
set(h,'linewidth',3);          % set linewith for all
set(h(2),'markersize',10);     % set markersize for 2nd plot

set(gca,'fontsize',20); % increase axis fontsize gca='[g]et [c]urrent [a]xis'
xlabel('Time or Step');
ylabel('Thousands of Dollars');
```