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Physics 35100 Mechanics
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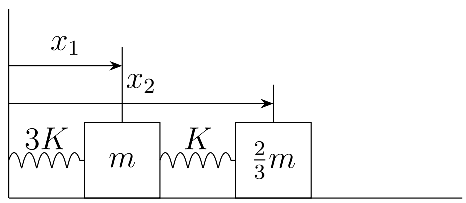
PSet 5

From *Classical Mechanics*, R. Douglas Gregory:

Chapter 4: 4.14, 4.15 (use Lagrangian Mechanics)

Question 1. *Double Oscillator* Two blocks of mass $m_1 = m$ and $m_2 = \frac{2m}{3}$ are resting at position x_1 and x_2 on a friction-less table. m_1 is connected to a linear spring with spring constant $K_1 = 3K$ and rest length L_1. m_2 is connected to a second linear spring with spring constant $K_2 = K$ and rest length L_2. Both masses are confined to move in one dimension.



(1) Write down the Lagrangian in the Cartesian coordinates x_1 and x_2.
(2) Find new generalized coordinates q_1 and q_2 such that all of the potential terms are of the form $V_k(q_k) = \frac{1}{2}Kq_k^2$. (Hint: $x_1 = q_1 + L_1$ will work for V_1.) Then rewrite the Lagrangian in the new coordinates.
(3) Find the two conjugate momenta p_1 and p_2.
(4) Find the equations of motion.
(5) The equations for the q_k are coupled making them difficult to solve. Show that a change of variables to $\dot{P}_1 = \dot{p}_1 + 2\dot{p}_2$ and $\dot{P}_2 = -2\dot{p}_1 + 3\dot{p}_2$ will uncouple the equations. For example, $\dot{P}_1 = \ddot{Q}_1 = -\Omega^2 Q_1$, where $Q_1 = 3q_1 + 2q_2$. Show that $Q_1 = 3q_1 + 2q_2$ and obeys $\ddot{Q}_1 = -\Omega_1^2 Q_1$ and find Q_2 and Ω_1 and Ω_2 in terms of the base frequency $\sqrt{\frac{K}{m}}$.

Question 2. *Matrix Lagrangian* Find the equation of motion for the Lagrangian:

$$L(\vec{q}, \dot{\vec{q}}) = \frac{1}{2}\vec{q}^T \mathsf{M} \vec{q} + \frac{1}{2}\dot{\vec{q}}^T \mathsf{K} \dot{\vec{q}},$$

where,

$$\vec{q}^T = \begin{bmatrix} \theta & \phi \end{bmatrix},$$

$$\mathsf{M} = \begin{bmatrix} 4\mu & 3\mu + \nu \\ 3\mu + \nu & \nu \end{bmatrix},$$

$$\mathsf{K} = \begin{bmatrix} 4\kappa & -\kappa \\ -\kappa & \kappa \end{bmatrix}.$$

Question 3. *Conservation of Energy* Consider a Lagrangian that obeys Euler-Lagrange equations. Show that the Hamiltonian will only be conserved $\dot{H} = 0$ if L does not explicitly depend on time $\frac{\partial L}{\partial t} = 0$. Consider a general Lagrangian $L(q_k(t), \dot{q}_k(t), t)$. (Hint: Find the total time derivative of the Lagrangian along with the Euler-Lagrange equations to relate terms to generalized momenta and their derivatives.)