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Physics 35100 Mechanics
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Problem Set 3

From *Classical Mechanics*, R. Douglas Gregory:
Chapter 6: 6.1, 6.5, 6.10

Question 1. *Impulse-Momentum Theorem* The impulse $\vec{J}(t_1, t_2)$ over the time interval t_1 to t_2 of a force $\vec{F}(t)$ is defined as:

$$\vec{J}(t_1, t_2) = \int_{t_1}^{t_2} \vec{F}(t)\, dt.$$

(1) Using Newton's Law for the momentum $\vec{p}$ of a particle of mass m and the fundamental theorem of calculus show that $\Delta\vec{p} = \vec{p}(t_2) - \vec{p}(t_2) = \vec{J}(t_1, t_2)$. This is the Impulse-Momentum Theorem: The change in momentum is equal to the Impulse.

(2) Show that the average force over the interval from t to $t + \Delta t$, $\vec{F}_{avg}$, times the size of the interval Δt is equal to the impulse $\vec{J}(t, t + \Delta t) = \vec{F}_{avg}\Delta t$, and therefore:

$$\frac{\Delta\vec{p}}{\Delta t} = \vec{F}_{avg}.$$

This discrete version of Newton's Law is not an approximation and is very useful when forces come in short bursts and otherwise there are no net forces.