

Topic F

Local evolution versus stationary action

Question and starting model

In what sense do initial-value and stationary-action descriptions specify the same mechanics, and where do their data differ? Establish the mathematics in one explicit system before drawing interpretive conclusions.

Use a harmonic oscillator with $m > 0$, $\omega > 0$, and

$$L(x, \dot{x}) = \frac{m}{2} \dot{x}^2 - \frac{m\omega^2}{2} x^2, \quad S[x] = \int_0^{T_f} L(x, \dot{x}) dt.$$

For an initial-value problem, specify $x(0) = x_a$ and $\dot{x}(0) = v_a$. For a fixed-endpoint variational problem, specify $x(0) = x_a$, $x(T_f) = x_b$, the duration T_f , and variations vanishing at the endpoints.

The word *stationary* means the first variation vanishes; it does not by definition mean that the action is minimized. A shared differential equation does not automatically imply identical data or a unique solution to every boundary-value problem.

What you need to start

The oscillator solution, Euler–Lagrange derivation, integration by parts, and careful reasoning about initial and endpoint data. No Hamilton–Jacobi theory, relativity, or general philosophy-of-time survey is required. Computation is optional.

Required foundation

Derive $\ddot{x} + \omega^2 x = 0$ from both the force law and the action. Keep the boundary term until its role is explained. Solve for a trajectory from initial data, then determine which initial velocity would connect prescribed endpoint positions over a prescribed duration.

Identify explicitly what is fixed and what is varied. Distinguish a property of the solution set from an additional claim about physical causation or explanation.

First-week task

Work through the endpoint equation for one ordinary duration and one duration with $\sin(\omega T_f) = 0$. Bring the equation relating the endpoint data to the initial velocity, and a concrete question about uniqueness or data sensitivity. This is the technical anchor of the proposal, not a preliminary essay.

Bounded investigation

First classify when fixed endpoint positions and duration select one oscillator trajectory, no trajectory, or more than one. Give explicit examples. Compare this with initial-value uniqueness.

Then choose **one** mathematical development:

1. **Sensitivity near an exceptional duration.** Quantify how a small change in an endpoint or the duration changes the inferred initial velocity. Use an analytic sensitivity or a small controlled numerical comparison and explain the conditioning.
2. **Stationary versus minimum.** Study the action difference for admissible variations $x(t) + \epsilon\eta(t)$, using $\eta(t) = \sin(n\pi t/T_f)$. Determine what the sign tells you about a minimum, degeneracy, or a saddle as the duration changes. A general calculus-of-variations classification is unnecessary.

Finally analyze **one precise interpretive claim**, such as “an endpoint in a variational problem implies that the future causes the past” or “a unique local evolution law must give a unique fixed-endpoint solution.” State the claim charitably, identify any extra premises, and give a serious alternative account or counterargument.

Validation and reading

Check proposed trajectories in the differential equation and against their stated data. Use explicit compatible and incompatible endpoint examples rather than a general assertion of equivalence. For numerical work, distinguish conditioning of the boundary problem from error in the solver.

Support specialist interpretive claims with appropriate reading. The resources sheet gives a starting discussion by Jeremy Butterfield; select the passages relevant to variational principles rather than treating an entire advanced paper as compulsory. An additional well-chosen source may be needed for the opposing argument. Cite what each source actually supports.

Stopping rule

A complete project contains the two mechanical derivations, the endpoint/initial-data comparison, one of the two mathematical developments, and one carefully argued interpretive discussion with a serious counterargument. Explain what the mechanics establishes and what remains interpretive. The quality of the reasoning, not the philosophical conclusion adopted, is graded.

Optional extension, only after completion

Compare the oscillator result with a discrete action or a free particle, or relate endpoint derivatives to later Hamilton–Jacobi material. An essay covering all theories of time or a general solution of the teleology debate is not required.

Starting references. Lectures 4–5 and the resources sheet. Your report should remain a mechanics investigation even when its final question is philosophical.

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