Graduate Classical Mechanics
Lecture 1 Instructor Outline
States, Dynamics, and the Goal of Physics

Lecture goal: Establish the conceptual frame for the course: complete state, deterministic evolution, projection to reduced descriptions, finite differences on edges, boundary structure, and physics as compression. Keep the philosophy concrete and move quickly toward mechanics.

Likely board content: about 5–6 boards/panels depending on room layout.

0. Opening question

ASK: What are we trying to accomplish when we do physics?

Take a few answers if useful: predict, explain, unify, model, control.

SAY: I want to organize the course around one additional answer: physics finds compact rules that generate enormous families of possible histories.

1. State and dynamics

BOARD:

$$\cdots \to S_{n-1} \to S_n \to S_{n+1} \to \cdots$$

BOARD:

$$\boxed{S_{n+1} = F(S_n)}$$

SAY: S_n means the complete state: whatever information is required for the evolution to close.

SAY: Do not read the index n as an ontological commitment yet. These may be fundamental discrete stages, or sampled points of a continuous flow. Lecture 3 will show both routes explicitly.

ASK: If I tell you only the position of a particle, have I specified its state?

Expected answer: not generally; need velocity or momentum as well.

BOARD:

$$q \quad \text{usually not closed}, \qquad S = (q, v) \text{ or } (q, p).$$

SAY: One of the deepest tasks in physics is not only finding F, but finding what belongs in S.

OPTIONAL: If useful, note that unique previous state requires F to be invertible:

$$S_n = F^{-1}(S_{n+1}).$$

Do not dwell unless asked.

2. Physics as compression

BOARD:

$$S_0, S_1, S_2, S_3, \ldots$$

SAY: We could describe the universe by listing all of its states. Complete, but almost no insight.

BOARD:

$$S_0 + F \quad \Longrightarrow \quad S_1, S_2, S_3, \ldots$$

BOARD:

$$\boxed{\text{history} \;\longrightarrow\; \text{initial state + law}}$$

SAY: This is enormous information compression. It gives prediction and technology, not merely a shorter description.

SAY: Course theme: Newton, Lagrange, and Hamilton are different ways to represent the dynamical rule and expose different structure.

3. Complete state versus reduced description

BOARD:

$$s_n = \mathsf{P}(S_n)$$

BOARD:

$$\begin{aligned} S_{n+1} &= F(S_n), \\ s_{n+1} &= \mathsf{P}(F(S_n)). \end{aligned}$$

SAY: There need not be a closed map $s_{n+1} = f(s_n)$ because the projection may have thrown away information.

BOARD:

$$\boxed{\mathsf{P} \circ F \neq f \circ \mathsf{P} \;\text{ in general}}$$

ASK: Classical example?

Use damped pendulum if needed.

BOARD:

$$S = (S_{\text{pendulum}}, S_{\text{environment}}), \qquad s = \mathsf{P}(S) = S_{\text{pendulum}}.$$

SAY: Apparent dissipation does not require the complete dynamics to be fundamentally dissipative. Energy can move into variables we chose not to keep.

OPTIONAL: QM, only if useful or asked:

$$|\Psi_{n+1}\rangle = U\,|\Psi_n\rangle\,.$$

Measurement can be discussed as correlations within the global state plus restriction to the branch/records accessible to the observer. Flag this as an Everett-like interpretation; do not turn the lecture into a QM foundations discussion.

4. States on nodes, changes on edges

BOARD:

$$\boxed{S_n} \xrightarrow{\Delta S_{n+1/2}} \boxed{S_{n+1}}$$

BOARD:

$$\Delta S_{n+1/2} = S_{n+1} - S_n.$$

SAY: The half-index is intentional. State lives on the node; first difference lives on the edge.

BOARD:

$$S_{n+1} = S_n + \Delta S_{n+1/2}.$$

ASK: If I tell you $\Delta q = 1\,\mathrm{m}$, do you know the new position?

Expected: no, need q_n.

SAY: A change is not a state. It relates two states.

5. FTC as telescoping and boundary

BOARD:

$$\begin{aligned}\sum_{n=M}^{N-1} \Delta S_{n+1/2} &= \sum_{n=M}^{N-1} (S_{n+1} - S_n) \\ &= (S_{M+1} - S_M) + (S_{M+2} - S_{M+1}) + \cdots \\ &= S_N - S_M.\end{aligned}$$

CHECK: Upper limit is $N - 1$ so final term is $S_N - S_{N-1}$.

BOARD:

$$\boxed{\sum \text{ local differences} = \text{boundary difference}}$$

SAY: This is the discrete FTC. Integration will later be the continuum version of accumulating changes.

SAY: More general theme: local changes in the interior become information on the boundary.

OPTIONAL: Mention generalized Stokes:

$$\int_\Omega \mathrm{d}\omega = \int_{\partial\Omega} \omega.$$

Say only that FTC, Green, divergence, and Stokes are versions of the same boundary principle.

6. Chain versus ring

BOARD: Draw 4–5 nodes in an open chain and then close them into a ring.

SAY: Open chain: endpoints survive.

BOARD:

$$\sum_{n=0}^{N-1} (S_{n+1} - S_n) = 0, \qquad S_N = S_0.$$

SAY: On the ring, there are N nodes and N edges. The "one fewer edge than node" observation belonged to an interval with a boundary, not to differentiation itself.

BOARD:

$$\boxed{\partial^2 = 0}$$

SAY: In ordinary language: the boundary of a boundary vanishes. We will see this same cancellation in higher-dimensional settings.

7. Second difference

BOARD:

$$\begin{aligned}
\Delta S_{n+1/2} &= S_{n+1} - S_n, \\
\Delta S_{n-1/2} &= S_n - S_{n-1}, \\
\Delta^2 S_n &= \Delta S_{n+1/2} - \Delta S_{n-1/2} \\
&= S_{n+1} - 2S_n + S_{n-1}.
\end{aligned}$$

CHECK: Centered formula: coefficient of S_n is -2.

BOARD:

$$\boxed{\text{nodes}} \xrightarrow{D} \boxed{\text{edges}} \xrightarrow{D^*} \boxed{\text{nodes}}.$$

SAY: A second-order equation is not just "two derivatives at a point." It compares edge information around a node and therefore naturally exposes neighboring states and boundaries.

NEXT: Next lecture: make this physical with masses and springs.

8. Time framing

If time allows, or earlier if discussion naturally goes there:

BOARD:

$$\boxed{\text{ordering of states}} \qquad \text{vs.} \qquad \boxed{\text{time measured by clocks}}$$

SAY: Simulation analogy: external processor speed need not affect the internal sequence of simulated states.

SAY: Inside the system, a clock is a repeating physical subsystem used to label change in other subsystems.

SAY: We are not settling the ontology of time; we are distinguishing state order from operational clock readings.

9. Close

BOARD: Leave these three boxes visible if possible:

$$\boxed{S_{n+1} = F(S_n)}$$

$$\boxed{s = \mathsf{P}(S)}$$

$$\boxed{\sum \Delta S = \text{boundary change}}$$

SAY: The semester will repeatedly ask three questions:

1. What is the state?
2. What rule evolves it?

3. What structure becomes visible in a particular formulation?

SAY: Final line: "The universe gives us histories. Physics searches for the states, constraints, and compact rules that generate them."

STOP: Natural endpoint. If running short, omit the detailed time discussion and QM aside; both are developed in the student notes.

Board checks / likely questions

- $S_{n+1} = F(S_n)$ gives a unique future. A unique past additionally requires invertibility of F.
- For the telescoping sum, use $n = M$ to $N - 1$.
- First difference is edge-centered: $\Delta S_{n+1/2}$.
- Centered second difference is $S_{n+1} - 2S_n + S_{n-1}$.
- Do not claim that objective-collapse models are ruled out; present the unitary/branch discussion as the conceptual viewpoint adopted here, not as settled interpretation.
- Do not claim that an external "time between updates" physically exists. The point is only that state ordering and operational clock time are conceptually distinct.