

Problem Set 7

Question 1. *Positive-Definite:* For the following symmetric matrices S , find S and determine if it is positive-definite, positive-semidefinite, or indefinite, and give a reason.

- (1) The 3×3 symmetric matrix S such that:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (x_1 + x_2 - 3x_3)^2$$

- (2) The 3×3 symmetric matrix S such that:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (x_1 + x_2 - 3x_3)^2 - x_1^2$$

- (3) $S = A^T A$ with

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$$

- (4) $S = A^T A$ with

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

Question 2. *Linear Least-squares:* The following data was obtained from an experiment:

x	0	1	2	3	4	5
y	1	21	100	240	439	701

- (1) Use MATLAB `polyfit` to fit the data to a quadratic: $y = ax^2 + bx + c$, and report the values of a , b , and c .
- (2) Use the normal form equation $A^T A \hat{u} = A^T y$ to find $\hat{u} = [a \ b \ c]^T$, where $y = [y_1 \ \dots \ y_N]^T = [1 \ \dots \ 701]^T$ is a column vector from the data above, and

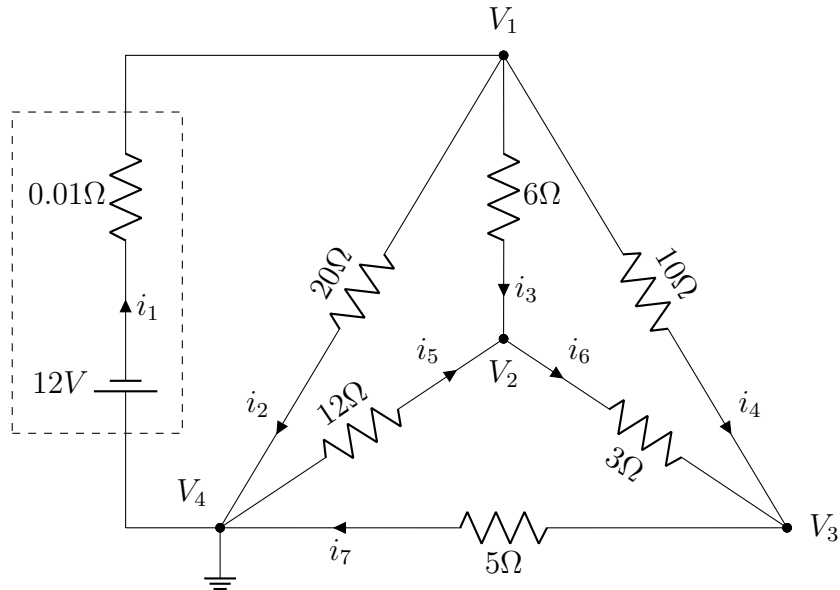
$$A = \begin{bmatrix} x_1^2 & x_1 & 1 \\ \vdots & \vdots & \vdots \\ x_N^2 & x_N & 1 \end{bmatrix}.$$

Compare \hat{u} to a , b , and c from `polyfit`.

- (3) One theory predicts that $c = 0$. Modify the the normal form equations to find the best fit to $y = ax^2 + bx$. The same theory predicts that $a = 30$ and $b = 10$. Does this fit provide evidence in favor of this theory?
- (4) Another theory predicts that $a = 28$, $b = 0$, and $c = 0$. Modify the the normal form equations to find the best fit to $y = ax^2$.

- (5) Plot the data with open circles and the fits (as lines) to the two theories $y = ax^2 + bx$ and $y = ax^2$ on the same plot. Which theory do you think is better and why?

Question 3. Network: Consider the following electric network:



- (1) Find the incident matrix A for the network above.
- (2) Use the framework:

$$u \xrightarrow{A,b} e = b - Au \xrightarrow{C} w = Ce \xrightarrow{A^T} A^T w = f,$$

to find the voltage of each node u and the current in each edge w .

- (3) (Kirchhoff's current law) Pick one node and show that the sum of the currents is zero.
- (4) (Kirchhoff's current law) Pick one edge and show that the voltage across the edge is equal to the current times the resistance in the edge.
- (5) Find the voltages and currents, if there is a 1 amp current source *into* the node at V_2 . Show that the sum of the currents into the node at V_2 is 1.