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Physics 39907 Computational Physics
November 14, 2024

Problem Set 7

Question 1. *Positive-Definite:* For the following symmetric matrices S, find S and determine if it is positive-definite, positive-semidefinite, or indefinite, and give a reason.

(1) The 3×3 symmetric matrix S such that:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (x_1 + x_2 - 3x_3)^2$$

(2) The 3×3 symmetric matrix S such that:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (x_1 + x_2 - 3x_3)^2 - x_1^2$$

(3) $S = A^T A$ with

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$$

(4) $S = A^T A$ with

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

Question 2. *Linear Least-squares:* The following data was obtained from an experiment:

x	0	1	2	3	4	5
y	1	21	100	240	439	701

(1) Use MATLAB `polyfit` to fit the data to a quadratic: $y = ax^2 + bx + c$, and report the values of a, b, and c.

(2) Use the normal form equation $A^T A\hat{u} = A^T y$ to find $\hat{u} = \begin{bmatrix} a & b & c \end{bmatrix}^T$, where $y = \begin{bmatrix} y_1 & \dots & y_N \end{bmatrix}^T = \begin{bmatrix} 1 & \dots & 701 \end{bmatrix}^T$ is a column vector from the data above, and

$$A = \begin{bmatrix} x_1^2 & x_1 & 1 \\ \vdots & \vdots & \vdots \\ x_N^2 & x_N & 1 \end{bmatrix}.$$

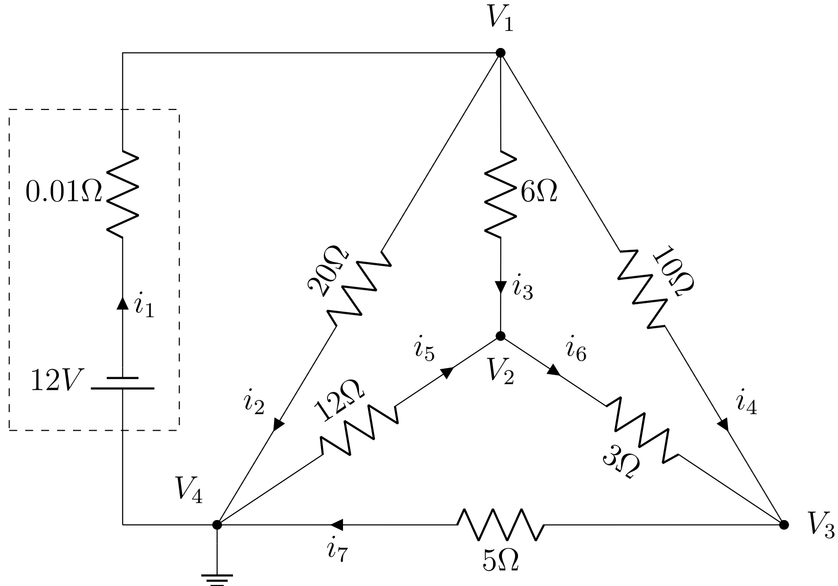
Compare $\hat{u}$ to a, b, and c from `polyfit`.

(3) One theory predicts that $c = 0$. Modify the the normal form equations to find the best fit to $y = ax^2 + bx$. The same theory predicts that $a = 30$ and $b = 10$. Does this fit provide evidence in favor of this theory?

(4) Another theory predicts that $a = 28$, $b = 0$, and $c = 0$. Modify the the normal form equations to find the best fit to $y = ax^2$.

(5) Plot the data with open circles and the fits (as lines) to the two theories $y = ax^2 + bx$ and $y = ax^2$ on the same plot. Which theory do you think is better and why?

Question 3. *Network:* Consider the following electric network:



(1) Find the incident matrix A for the network above.

(2) Use the framework:

$$u \xrightarrow{A,b} e = b - Au \xrightarrow{C} w = Ce \xrightarrow{A^T} A^T w = f,$$

to find the voltage of each node u and the current in each edge w.

(3) (Kirchhoff's current law) Pick one node and show that the sum of the currents is zero.

(4) (Kirchhoff's current law) Pick one edge and show that the voltage across the edge is equal to the current times the resistance in the edge.

(5) Find the voltages and currents, if there is a 1 amp current source *into* the node at V_2. Show that the sum of the currents into the node at V_2 is 1.