Problem Set 3

From *Classical Mechanics*, R. Douglas Gregory:

**Chapter 6:** 6.1, 6.5, 6.10

**Question 1. Impulse-Momentum Theorem** The impulse \( \vec{J}(t_1, t_2) \) over the time interval \( t_1 \) to \( t_2 \) of a force \( \vec{F}(t) \) is defined as:

\[
\vec{J}(t_1, t_2) = \int_{t_1}^{t_2} \vec{F}(t) \, dt.
\]

(1) Using Newton’s Law for the momentum \( \vec{p} \) of a particle of mass \( m \) and the fundamental theorem of calculus show that \( \Delta \vec{p} = \vec{p}(t_2) - \vec{p}(t_1) = \vec{J}(t_1, t_2) \). This is the Impulse-Momentum Theorem: The change in momentum is equal to the Impulse.

(2) Show that the average force over the interval from \( t \) to \( t + \Delta t \), \( \vec{F}_{avg} \), times the size of the interval \( \Delta t \) is equal to the impulse \( \vec{J}(t, t + \Delta t) = \vec{F}_{avg} \Delta t \), and therefore:

\[
\frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{avg}.
\]

This discrete version of Newton’s Law is not an approximation and is very useful when forces come in short bursts and otherwise there are no net forces.