

First-order Differential Equations

$$y' = ay + q(t)$$

$q(t)$	$y(t)$ given $y(0)$	$y(t)$
0	$y(0)e^{at}$	Ce^{at}
q	$y(0)e^{at} + \frac{q}{a}(e^{at} - 1)$	$Ce^{at} - \frac{q}{a}$
qe^{st}	$y(0)e^{at} + \frac{q}{s-a}(e^{st} - e^{at})$	$Ce^{at} + \frac{q}{s-a}e^{st}$
qe^{at}	$y(0)e^{at} + qte^{at}$	$Ce^{at} + qte^{at}$
$q(t)$	$y(0)e^{at} + \int_0^t e^{a(t-s)}q(s) ds$	$e^{at} \left(C + \int_0^t e^{-as}q(s) ds \right)$
$q\delta(t-T)$	$y(0)e^{at} + qH(t-T)e^{a(t-T)}$	$Ce^{at} + qH(t-T)e^{a(t-T)}$
$qH(t-T)$	$y(0)e^{at} + qH(t-T)(e^{a(t-T)} - 1)$	$Ce^{at} + qH(t-T)e^{a(t-T)}$
$qe^{i\omega t}$	$y(0)e^{at} + \frac{q}{i\omega - a}(e^{i\omega t} - 1)$	$Ce^{at} + \frac{q}{i\omega - a}e^{i\omega t}$
$q(t)$ & $a(t)$	$y(0)e^{\int_0^t a(T) dT} + \int_0^t e^{\int_s^t a(T) dT}q(s) ds$	$Ce^{\int_0^t a(T) dT} + \int_0^t e^{\int_s^t a(T) dT}q(s) ds$