Comment on "Spontaneous Wave Pattern Formation in Vibrated Granular Materials"

Molecular dynamics simulations by Aoki and Akiyama (AA) [1] of vertically oscillated two-dimensional (2D) granular beds yielded standing wave patterns with wavelengths in qualitative accord with those found in 3D [2] and 2D [3] experiments. This result is remarkable in light of several nonphysical properties of AA's particles. AA's nondimensionalization leads to a dependence of particle properties upon the oscillation frequency. In addition, the form of particle interaction used by AA gives nonphysical behavior for the restitution coefficient, which leads them to conclude that tangential friction is necessary for pattern formations, contrary to simulations by ourselves and in [4]. Although not mentioned in [1], AA's patterns form for an acceleration amplitude far below that observed in experiments.

AA simulate particles in a box with a bottom that oscillates vertically with frequency f. The particles interact via a modified 6-12 potential and a velocity-dependent dissipation. To enforce a fixed number of time steps per oscillation, AA nondimensionalize with the time scale $1/f$ and the length scale in the potential d [1], so that the box bottom moves with $z' = (A/d)\sin(2\pi t')$, where primed variables are dimensionless. The equation of motion for the position $\mathbf{r}_i$ of the ith particle becomes

$$\ddot{\mathbf{r}}'_i = \sum_{j\neq i}\left[\frac{-\epsilon}{md^2f^2}\frac{\partial\phi'}{\partial\mathbf{r}'_{ij}} - \sum_{\alpha=n,t}\frac{\gamma_\alpha \mathbf{v}'_{ij,\alpha}}{f}\right] + \frac{\mathbf{g}}{df^2}, \quad (1)$$

where m is the particle mass, $\epsilon\phi'$ is the potential, γ_n and γ_t are coefficients for the dissipation in the normal and tangential directions, $\mathbf{g}$ is the gravitational acceleration, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{v}_{ij,n}$ and $\mathbf{v}_{ij,t}$ are the normal and tangential components of the relative velocity, and the sum on j extends over all particles within $D = 2^{1/6}d$ of particle i. AA define a control parameter $g_s = g/df^2$; hence, the first two terms on the right-hand side of (1) have prefactors $(\epsilon/mgd)g_s$ and $\gamma_\alpha\sqrt{dg_s/g}$. AA hold these prefactors constant as they vary g_s; thus, in their model, particle properties are frequency dependent.

We have calculated the restitution coefficient e for AA's potential and radial dissipation term and have found that e approaches 1 with increasing collision velocity, rather than decreasing as found in the experiments [5]. This behavior has also been found [5] for the other potential used by AA, a damped Hertzian spring. As a consequence, AA state that tangential dissipation is *necessary* for pattern formation, but Luding [4] and ourselves obtain patterns in simulations with no tangential dissipation; as in [4], we only assume radially dissipative hard sphere interactions, momentum conservation, and free fall of particles between collisions. Because AA's radial dissipation is inefficient, they require tangential dissipation to remove the energy given to the particles via vibration.

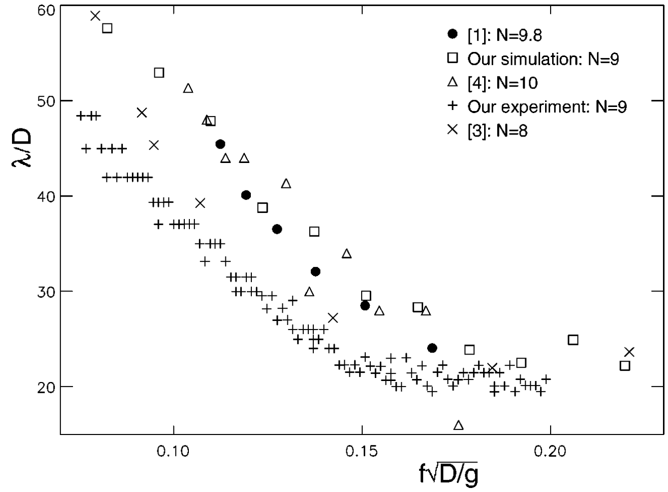

FIG. 1. Dimensionless wavelength vs frequency, where N is the number of particle layers and D is the particle diameter.

In our experiments and simulations and those in [2–4], patterns form when the dimensionless acceleration amplitude $\Gamma = (2\pi f)^2 A/g$ exceeds about 2.5. AA present enough information to calculate $\Gamma = (2\pi)^2 A/dg_s$ only for their Fig. 1 in [1], for which $\Gamma = 1.2$, less than half the observed onset value.

The wavelengths λ obtained from 2D experiments and simulations are displayed in Fig. 1. The level of agreement is surprisingly good, given the differences in simulation models, the dependence of particle properties on frequency in AA's simulation, and the different materials used experimentally (Al in [3] and Pb in our experiments). This suggests that once patterns form, the macroscopic behavior of the medium is not strongly dependent on the details of the microscopic particle interactions.

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