2D Vertically Vibrated Granular Media: Experiment and Simulation

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ABSTRACT: We report on laboratory experiments and numerical simulations of a two-dimensional (2D) vertically oscillated granular medium. The experiment consists of a thin evacuated cell filled with 0.46 ± 0.03 mm lead particles vibrated sinusoidally ($A\sin(2\pi ft)$) in the vertical direction. The frequency, f, and the peak acceleration are the experimental control parameters. The event-driven simulation [2] reproduces all of the qualitative features the experimental observations. Specifically, as The peak acceleration is increased slightly above $1g$, the layer of particles leaves the bottom of the cell for part of the cycle, but the surface remains flat. When the peak acceleration reaches a critical value of approximately $2.5g$, the flat layer loses stability to a sub-harmonic standing wave. The wavelength and shape of the standing wave are similar in both the experiment and simulation. As the frequency is lowered, at constant peak acceleration, the amplitude of the standing wave becomes greater and its wavelength becomes longer.

1 INTRODUCTION

On a microscopic scale, granular dynamics are governed by highly dissipative contact interactions. Even though all interaction are short-range experiments demonstrate both system-wide [7, 8] and localized (30-100 particle) [10] collective behavior in vertically vibrated evacuated wide containers of granular material. A similar system-wide collective behavior is seen in thin (2D) containers as well [3]. Event-driven binary collision model simulations [5] and molecular dynamics simulations [1] of 2D systems qualitatively reproduce the results of these experiments. In this work we reproduce the results of Luding et al. and Clément et al. for a larger aspect ratio cell in preparation for future studies of the microscopic properties of these systems. The 2D experimental system and simulations provide an excellent opportunity to probe the microscopic behavior of these systems.

2 EXPERIMENT

The experiments consist of a thin parallelepiped cell filled with 9 layer of lead particles of mean diameter 0.46 ± 0.03 mm, which is about 15 percent less than the thickness of the cell. The cell shown in figure 1 has dimensions of 284x0.533x95 mm or 616x1.16x206 particles diameters and is oriented with the longest and the shortest sides horizontal and vibrated sinusoidally ($Asin(2\pi ft)$) in the vertical direction by an industrial electro-mechanical shaker. The physical control parameters are the amplitude, A, which can vary up to 1 cm, and the frequency, f, which we have varied from 10-100 Hz. Experiments are typically performed at constant, $\Gamma = A(2\pi f)^2/g$ and the frequency of oscillation, f, is varied. $\Gamma$is a dimensionless measure of the acceleration with respect to the magnitude of the acceleration due to gravity, g.

One face of the cell is transparent for visualization by a high speed digital camera (see figure 1). The camera has a frame rate of 228 fps, and an acquisition matrix of 256x256 pixels. Using this camera we acquire movies of the pattern formation at 228 fps or we synchronize the camera frame rate with the shaker frequency. Using these images we can compare the experiment with the simulation.

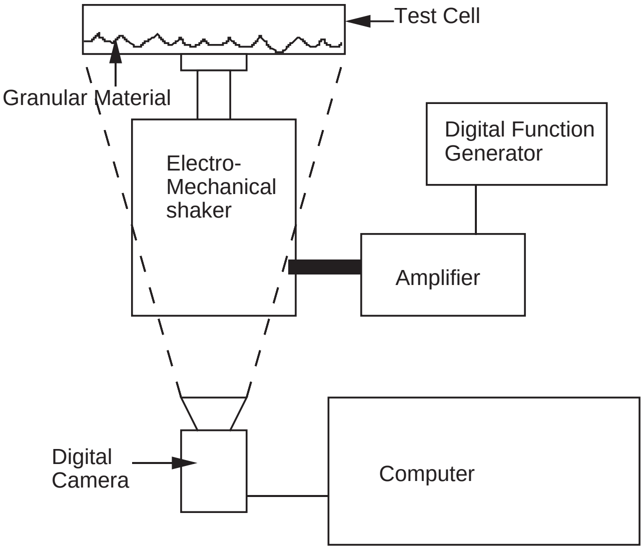

Figure 1: Schematic of the experimental apparatus showing the test cell, the shaker, and the imaging system.

3 SIMULATION

The simulation employs an event-driven inelastic binary collision model [9, 6, 4]. In this type of simulation time advances from collision to collision using ballistic motion. A sorted list of the time-to-next-collision is maintained for each particle and is used to determine the next collision. The collision duration is assumed zero, therefore limiting the particle interactions to binary collisions. Rotations are not included. The particles have a distribution of diameters, but theirs masses are equal. Momentum conservation and a collision-velocity dependent coefficient of restitution, $\epsilon(v_c)$, are used to calculate $\mathbf{v}_1'$ and $\mathbf{v}_2'$, the velocities after a particle-particle collision, from $\mathbf{v}_1$ and $\mathbf{v}_2$, the velocities before a particle-particle collision.

$$v_{n\substack{1\\2}} = \mathbf{v}_{\substack{1\\2}} \cdot \hat{n} \qquad (1)$$

$$v_{n1}' = \frac{1}{2}(1-\epsilon)v_{n1} + (1+\epsilon)v_{n2} \qquad (2)$$

$$v_{n2}' = \frac{1}{2}(1-\epsilon)v_{n2} + (1+\epsilon)v_{n1} \qquad (3)$$

$\hat{n}$ is a unit vector in the direction of a line connecting the centers of the colliding balls. In collisions with the walls, the wall is treated like a particle of infinite mass, and a tangential friction, μ, is added:

$$v_t = |\mathbf{v} - v_n\hat{n}| \qquad (4)$$

$$v_t{}' = \max(v_t - \mu v_n, 0) \qquad (5)$$

μ is set to .1 for all simulations in this paper.

The simulation is 3D and simulates the 2D geometry using closely spaced walls, just as in the experimental case. The simulation is non-dimensionalized using D, the diameter of the particles, and $\sqrt{g/D}$ for time. Thus, for comparisons experimental parameter must be expressed in terms of these scalings. Γ is already dimensionless and we define a non-dimensional frequency, $f_0 = f\sqrt{D/g}$.

4 RESULTS

The simulations and the experiments give similar results. In both, as Γ is increased slightly above 1, the layer of particles begins to leave the bottom of the cell for part of the cycle, but the surface remains flat. As Γ is increased to a critical value, $\Gamma_c \simeq 2.5$, the flat layer loses stability to a sub-harmonic standing wave, as shown in figure 2, which shows the experiment on the left and simulation on the right for $\Gamma = 3.0$ and $f_0 = .14$. For the particles used in these experiments the dimensional frequency is $f = 20.386$ Hz. This figure shows four snapshots of the entire cell during two cycles of the driving frequency which is one cycle of the pattern. The shape of the waves and the time course are captured in the simulation. There is a difference in the absolute wavelength and amplitude, but the dependence of the wavelength and amplitude on f_0 are consistent as shown in figure 3. Figure 3 shows a single snapshot of the pattern at four different $f_0 = .082, .94, .123, .140$ for the experiment and the simulation. The wavelength and the amplitude of the pattern decreases with increasing frequency.

5 ACKNOWLEDGMENTS

This research is supported by the U.S. Department of Energy Office of Basic Energy Sciences and the Texas Advanced Research Program.

References

[1] Keiko M. Aoki and Tetsuo Akiyama. Spontaneous wave pattern formation in vibrated granular materials. *Phys. Rev. Lett.*, 77(20):4166–4169, 1996.

[2] C. Bizon, M. D. Shattuck, J. B. Swift, W. D. McCormick, and Harry L. Swinney. Pat-

Experiment Simulation

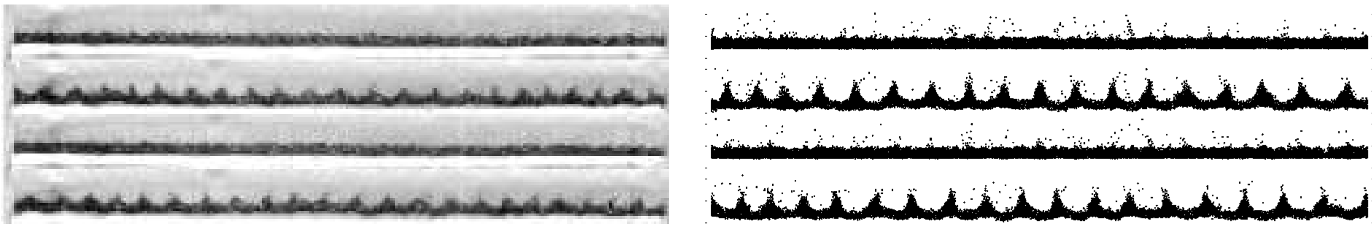

Figure 2: Time course of sub-harmonic pattern for experiment (left) and simulation (right) at $\Gamma = 3.0$ and $f_0 = .10$. Four time steps are shown. Starting from the top $t = .28$ T, $t = .78$ T, $t = 1.28$ T, $t = 1.78$ T, where T is the period of drive oscillation.

Experiment Simulation

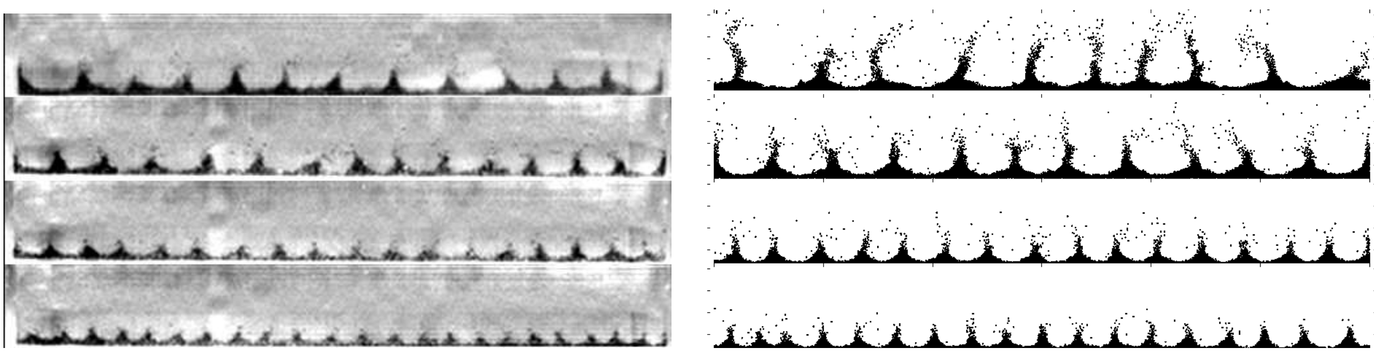

Figure 3: Frequency dependence of sub-harmonic pattern for experiment (left) and simulation (right) at $\Gamma = 3.0$. Frequency increases from the top frame to bottom frame in the figure. Starting from the top $f_0 = .082$, $f_0 = .94$, $f_0 = .123$, and $f_0 = .140$.

terns in 3D vertically oscillated granular layers: Simulation and experiment. *Phys. Rev. Lett.*, 80(1):57–61, 1998.

[3] Eric Clément, Loic Vanel, Jean Rajchenbach, and Jacques Duran. Pattern formation in a vibrated granular layer. *Phys. Rev. E.*, 53(3):2972–2975, 1996.

[4] S. Luding, E. Clément, A. Blumen, J. Rajchenbach, and J. Duran. Anomalous energy dissipation in molecular-dynamics simulations of grains: The "detachment effect". *Phys. Rev. E*, 50(5):4113–4120, 1994.

[5] S. Luding, E. Clément, J. Rajchenbach, and J. Duran. Simulations of pattern formation in vibrated granular media. *Europhys. Lett.*, 36(4):247–252, 1996.

[6] Mauricio Marín, Dino Risso, and Patricio Cordero. Efficient algorithms for many-body hard particle molecular dynamics. *J. Comput. Phys.*, 109:306–317, 1993.

[7] Francisco Melo, Paul Umbanhowar, and Harry L. Swinney. Transition to parametric wave patterns in a vertically oscillated granular layer. *Phys. Rev. Lett.*, 72:172–175, 1994.

[8] Francisco Melo, Paul B. Umbanhowar, and Harry L. Swinney. Hexagons, kinks, and disorder in oscillated granular layers. *Phys. Rev. Lett.*, 75(21):3838–3841, 1995.

[9] D. C. Rapaport. The event scheduling problem in molecular dynamics simulation. *J. Comput. Phys.*, 34:184–201, 1980.

[10] Paul Umbanhowar, Francisco Melo, and Harry L. Swinney. Localized excitations in a vertically vibrated granular layer. *Nature*, 382:793–796, 1996.